DIRECTIONAL CHARACTERISTICS OF THE 1990-1999 WAVE INFORMATION STUDIES GULF OF MEXICO HINDCAST

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1. INTRODUCTION

Wave hindcast information is invaluable to coastal engineers, and validation of this hindcast information is of utmost importance. The usual validation procedure is to compare hindcast information with measured data at a coincident location. Wave height and period can be validated easily using linear statistics but validation of wave direction is more complex since we are dealing with circular data. This paper discusses statistical measures that give insight into the comparison of hindcasted mean wave direction with measured mean wave direction. The statistical procedures described in this paper could be applied to any of the other directional parameters available from the hindcast. Results are shown for the recent 1990-1999 Wave Information Studies (WIS) Gulf of Mexico Level 3 Wave Hindcast produced at the Engineer Research and Development Center's Coastal and Hydraulics Laboratory at the Waterways Experiment Station in Vicksburg, Mississippi.

2. HINDCAST BACKGROUND INFORMATION

WIS has produced wave hindcast information for all the coastlines of the United States, and Gulf of Mexico hindcast information has been created for 1956-1975 and for 1976-1995. A new hindcast has recently been completed for 1990-1999 to take advantage of more Gulf of Mexico wind and wave measurement locations, improved wind fields, and refinements in the wave hindcasting model, WISWAVE. WISWAVE is a second generation spectral finite depth wave model which models the physics of wave generation from a wind source function and uses a finite difference propagation scheme to move wave energy on a set of rectangular grid points based on latitude and longitude. WISWAVE was developed by Dr. Don Resio; and Resio (1981,1982), Resio and Tracy(1983) and Hubertz (1992) describe the model. Quality wind fields are an essential component in the hindcasting process. The 1990-1999 wind fields were purchased from Oceanweather, Inc. These wind fields were created by assimilating satellite information into the National Centers for Enviromental Prediction (NCEP) 6-hour wind fields available from NOAA. Measured wind data and tropical storm information were assimilated and blended to produce hourly wind fields. The final wind field product utilizes all available information and expert meteorological analysis. The Gulf of Mexico hindcast included three levels, a 1-degree grid that covered the Atlantic and Gulf, a Level 2 1/4-degree grid that covered the Gulf of Mexico, and a Level 3 1/12-degree grid that covered the entire Gulf coastal area. Each grid received spectral boundary information from the previous level.

The Gulf hindcast information was compared to all available measurements in the area. Several measurement devices in the Gulf were directional in the late 1990's and were used to validate the quality of the hindcasted wave directional information. These directional measurement sites are shown in Figure 1. The grid in Figure 1 corresponds to the domain of the Level 3 Gulf grid. Table 1 lists relevant information on these measurement sites.

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Gulf of Mexico with NDBC Directional Buoys

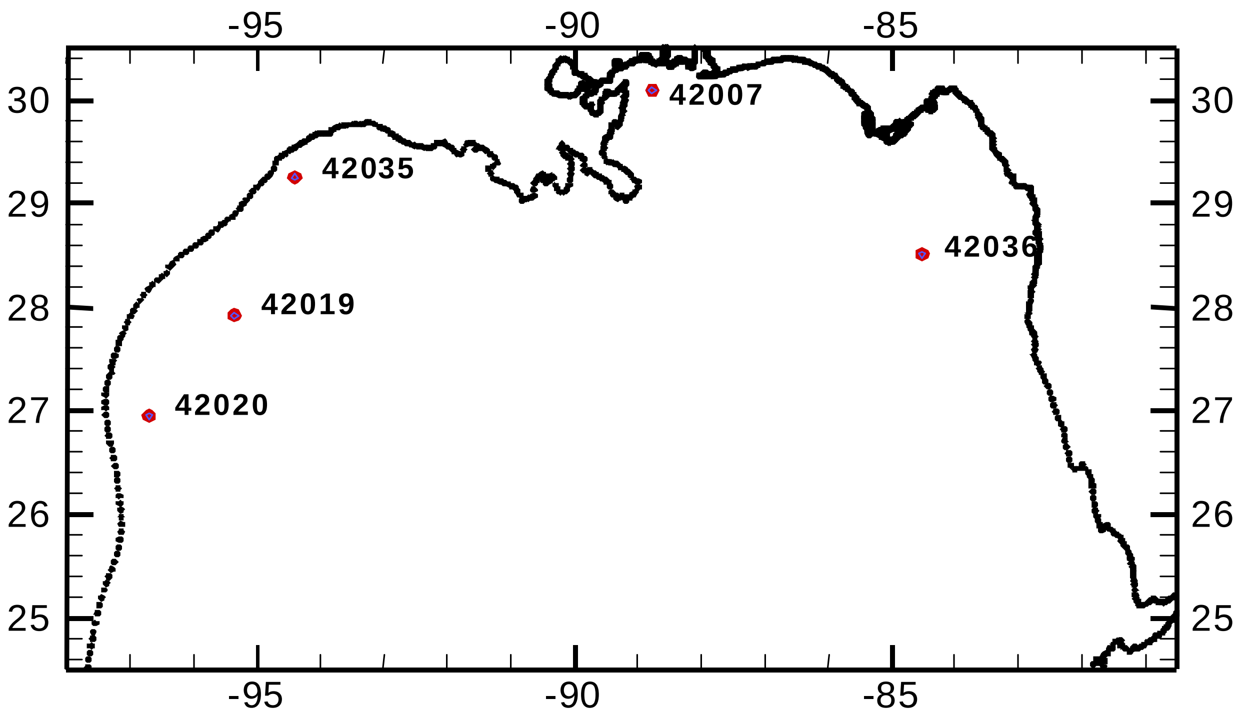

Figure 1. Gulf of Mexico directional measurement stations showing the domain of the Level 3 Gulf hindcast

TABLE 1. NDBC Directional Buoys

Buoy Number	Latitude N	Longitude W	Location	Depth (m)
42020	26.95	96.70	Corpus Christi, TX	78.6
42019	27.92	95.36	Freeport, TX	82.3
42035	29.25	94.41	Galveston, TX	15.9
42007	30.09	88.77	Biloxi, MS	13.4
42036	28.51	84.51	W. Tampa, FL	53.0

3. STATISTICS RELATED TO DIRECTIONAL DIFFERENCE

Linear statistics give information on the energy-based quantities such as wave height but comparisons involving wave directions between the WIS hindcast information and the measured information require circular statistics. Circular statistical quantities using the vector mean wave direction were calculated for the coincident WIS and measurement locations for the 1990 Gulf of Mexico hindcast. The mean directional difference between WIS and the measurement location was calculated for one month of data (using one month of information helps to identify seasonal trends and identifies differences in specific storms during the month). Monthly statistics give a measure of how one month's hindcast information compares with what has been measured, and these statistics can be added to the monthly statistics already developed for parameters like significant wave height and period. Concentration and circular correlation, two other statistics using the monthly set of directional differences give more information about how well the measured and hindcasted mean directions compare. The concentration statistic measures the spread of the distribution. The circular correlation, similar in concept to the usual linear correlation coefficient but applied to circular data, measures how the two distributions match. It is important to look at all three of these statistics since one month could show a small mean directional difference but could have low circular correlation and a concentration value indicating a distribution with a lot of spread. The following paragraphs present the statistical equations and variables used in the tables and graphs in section 4 on statistical results.

All wave directions for both WIS and the measurements are initially in meteorological convention. These values are converted to polar coordinates for ease in dealing with trigonometric functions. The directional difference, x_i, is calculated for each hour that has a coincident directional measurement and WIS value. The WIS direction and the measurement direction are assumed to be two unit vectors in polar coordinates. Using the scalar and cross vector product on these two vectors enables calculation of a directional difference (positive or negative). This difference was calculated so a positive difference means that the measurement vector is more clockwise than the WIS wave direction vector (using polar directions). An example of a directional difference calculation will help to clarify the results. If the measured wave direction is 18.9 deg in meteorological convention (coming from NNE) and the WIS wave direction is 57.9 deg in meteorological convention (from NE), the calculated difference in the statistical program used for the statistics in this paper will be –39.0 deg. The mean directional difference is calculated from the set of directional differences using the technique outlined in Bowers et al. (2000) to calculate $\bar{x}$, the mean directional difference for the month in question. Equations 1 through 5 below show the steps in the calculation of $\bar{x}$, the mean directional difference.

$$S = \sum_{i=1}^{n} \sin(x_i) \tag{1}$$

$$C = \sum_{i=1}^{n} \cos(x_i) \tag{2}$$

$$R = \sqrt{C^2 + S^2} \tag{3}$$

$$\bar{R} = R/n \tag{4}$$

$$\bar{x} = \tan^{-1}(S/C) \tag{5}$$

R is called the resultant, and $\bar{R}$ is termed the mean resultant length since it is divided by n, the number of coincident directions in the two distributions. The arc tangent function was calculated using the FORTRAN atan2 function to assure proper signs and quadrants for the resulting $\bar{x}$ angle. The 95% confidence interval for $\bar{x}$ can be obtained using

$$\bar{x} \pm \sin^{-1}(1.96/\sqrt{n\bar{R}\hat{k}}) \tag{6}$$

where $\hat{k}$ is the approximate concentration defined by the expressions below from Bowers et al.(2000) using the mean resultant defined above:

$$\hat{k} = 2\overline{R} + \overline{R}^3 + 5\overline{R}^5/6 \qquad \text{if } \overline{R} < 0.53$$
$$\hat{k} = -0.4 + 1.39\,\overline{R} + 0.43(1-\overline{R}) \qquad \text{if } 0.53 \le \overline{R} < 0.85 \tag{7}$$
$$\hat{k} = 1/(\overline{R}^3 - 4\overline{R}^2 + 3\overline{R}) \qquad \text{if } \overline{R} \ge 0.85$$

The concentration statistic is a measure of the tightness of the distribution. If the concentration is >5.0, the distribution is tightly grouped; lower concentration values indicate a distribution with more spread Bowers et al.(2000).

Correlation statistics involving the two distributions are also helpful is defining whether the measurement mean wave direction distribution and the WIS mean wave direction distribution are similar for the month of data. This correlation in addition to the concentration statistic adds credibility to the mean directional difference. The circular correlation defined in Bowers et al. (2000) and Fisher (1993) was used in addition to an adaptation of the usual linear correlation coefficient, *cor*, defined in Johnson and Wichern (1999). The circular correlation, *circor*, is defined by

$$circor = \frac{\sum_{1 \le i \le j \le n} \sin(x_i - x_j)\sin(y_i - y_j)}{\sqrt{\sum_{1 \le i \le j \le n} \sin^2(x_i - x_j) \sum_{1 \le i \le j \le n} \sin^2(y_i - y_j)}} \tag{8}$$

where x represents measurement directions and y represents WIS directions. The *i* and *j* subscripts refer to the various hourly measurements that range from 1 to n. The linear correlation coefficient, *cor*, using each distribution's mean directional difference was also used to give information on the relationship of the two distributions. In this representation, sine functions of the angular differences were used rather than just angular differences.

$$cor = \frac{\sum_{i=1}^{n} \sin(q_m - \overline{q}_m)\sin(q_w - \overline{q}_w)}{\sqrt{\sum_{i=1}^{n} (\sin(q_m - \overline{q}_m))^2 \sum_{i=1}^{n} (\sin(q_w - \overline{q}_w))^2}} \tag{9}$$

$\overline{q}$ in Equation 9 is the mean direction corresponding to each of the distributions; the *m* subscript corresponds to the measurement distribution and the *w* subscript corresponds to the WIS direction distribution. Numerical values for circular correlation and linear correlation were very close with the circular correlation sometimes being more sensitive and giving a slightly lower correlation estimate than the linear statistic.

The mean resultant length defined above can be used to define the sample circular variance (Fisher, 1993), *V*, by

$$V = 1 - \overline{R} \tag{10}$$

The sample circular standard deviation is defined as (Fisher, 1993):

$$v = \left[-2\log(1-V)\right]^{1/2} \tag{11}$$

which can be approximated by

$$v \cong (2V)^{1/2} \text{ for } V \text{ small}$$
$$v \cong \left[2(1-\overline{R})\right]^{1/2} \text{ for } \overline{R} \text{ large} \qquad (12)$$

4. STATISTICAL RESULTS

Monthly statistical calculations were done for each of the months that had wave direction measurements from 1990-1999. The overall vector mean wave direction parameter was used in these calculations both for the WIS information and the measurement information. Statistics are presented for waves 1.0 m and higher. The measurement height was used for imposing the 1-m limit. Directions of low wave heights can diverge and contaminate the directional difference distribution and the 1-m limit produced consistent results for the various locations.

Figure 2 shows the wave direction statistics for NDBC 42007 and WIS for 1997 excluding August, which had less than 20 coincident direction pairs. Figure 2 also excludes January through March because there were no measurements. The monthly mean directional difference is shown by a circle. Note that most months except April, May, and December (months 4, 5, and 12) show positive mean directional differences indicating that the buoy wave direction (in polar coordinates using vector direction-waves going toward this direction) is more clockwise than the WIS direction. The maximum mean directional difference, 16 deg, occurs in September, 1997. The concentration statistic is shown by a triangle in Figure 2. Recall that concentrations below 5.0 indicate more dispersion in the distributions. The May (month 5) results fall in this category. Correlations for all months fall between 0.75 and 0.90 indicating reasonable correlation between the measured and hindcasted wave directions. Another very important factor in the statistics is the number of coincident observations. May has only 49 observations so this may not be a good sample of the whole month. Figure 3 shows a plot of the 118 directional differences for November. The plot shows a definite bias to positive directional differences. An event right after November 20 shows some obvious disagreement between 42007 and WIS. Figure 4 gives a picture of the actual comparison by showing measured mean wave directions at 42007 plotted with coincident WIS mean wave directions for November 1997. Note that wave directions in Figure 4 are in meteorological convention. July has a small directional difference, concentration above 5.0, and 0.966 correlation (see Figure 2). Figure 5 shows the measured and WIS information that produced these statistics. Note that all these plots show that the concentration statistic and circular correlation provide an excellent description of the monthly directional comparison. Table 2 lists all the statistical values for the comparisons of mean wave direction for NDBC 42007 and WIS for 1997. Column 3 lists the monthly mean directional difference in degrees; columns 4 and 5 give the upper and lower 95% confidence limits on the monthly mean directional difference; $\hat{k}$ is the concentration statistic. The number of paired values for the month is shown in the next column; *cor* is the linear correlation; *circor* is the circular correlation. The last 3 columns give the resultant, variance and standard deviation of the directional difference distribution. A researcher could take this statistical information into account and verify differences in results from using WIS directional hindcast information at this time and location.

Table 2. Buoy 42007											
Year	Month	$\bar{X}$ (deg)	$\bar{X}$ -95% confidence (deg)	$\bar{X}$ +95% confidence (deg)	$\hat{k}$	Number	Cor	Circor	Resultant	Variance	Standard deviation
1997	4	-2.6	-0.11	-5.08	9.25	234	0.874	0.847	0.944	0.056	0.334
1997	5	-10.27	-0.67	-19.88	3.37	49	0.865	0.782	0.835	0.165	0.574
1997	6	10.6	14.2	7	16.2	62	0.775	0.772	0.969	0.031	0.251
1997	7	1.52	5.91	-2.87	15.08	45	0.967	0.896	0.966	0.034	0.26
1997	8	42.99	45.92	40.06	184.16	8	0.857	0.857	0.997	0.003	0.074
1997	9	16.11	19.53	12.68	20.45	54	0.883	0.868	0.975	0.025	0.223
1997	10	13.61	16.21	11.01	6.93	291	0.836	0.791	0.925	0.075	0.388
1997	11	7.32	11.51	3.13	6.62	118	0.805	0.784	0.921	0.079	0.397
1997	12	-1.46	2.31	-5.22	7.53	127	0.539	0.793	0.931	0.069	0.371

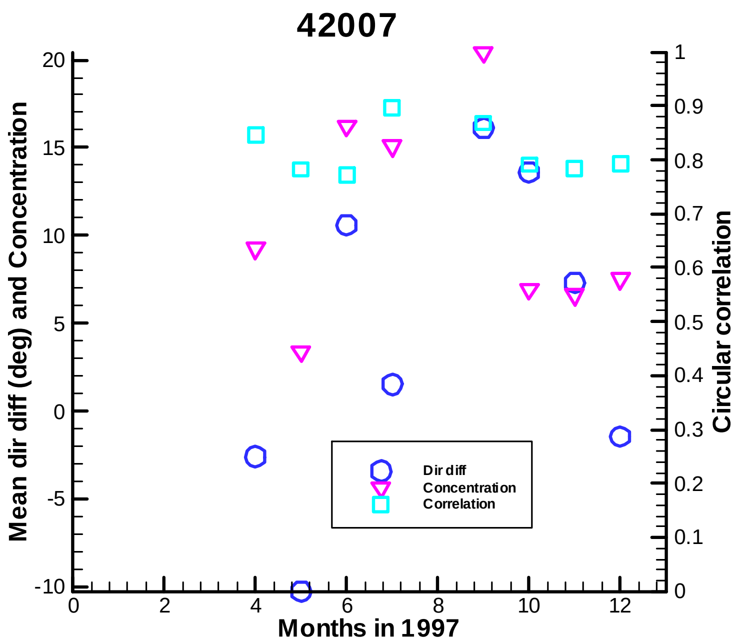

Figure 2. 1997 directional statistics for 42007 and WIS. Mean directional difference and concentration are plotted on the left-hand y-axis and circular correlation is plotted on the right-hand y-axis.

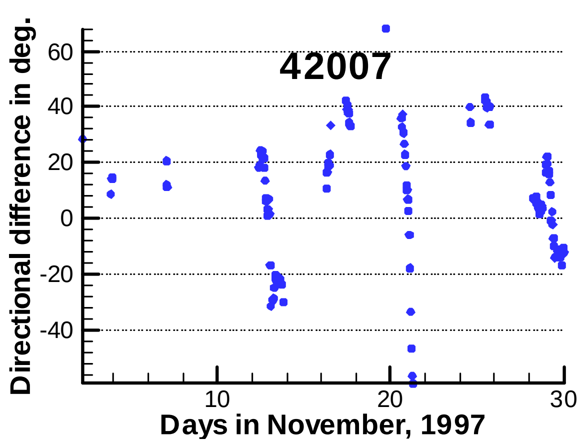

Figure 3. Directional differences in degrees for 42007 in November, 1997.

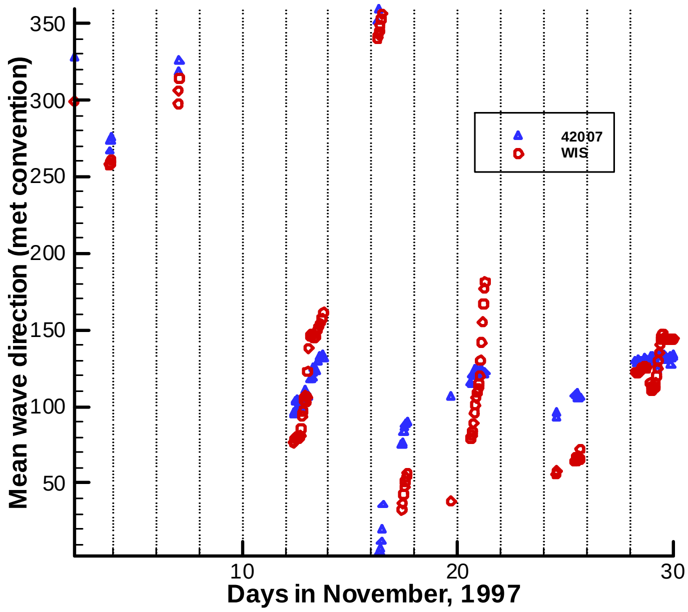

Figure 4. Measured and WIS mean wave directions in degrees for location at 42007 in November, 1997. Only directions for waves greater than or equal to 1 m are included.

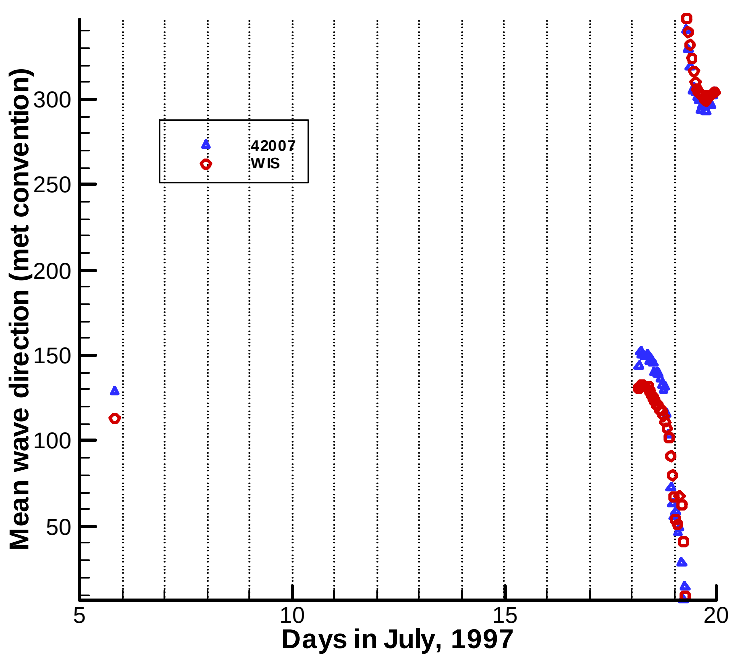

Figure 5. Measured and WIS mean wave directions in degrees for location at 42007. Only directions for wave greater than or equal to 1 m are included.

Figure 6 shows the statistics for the directional location at 42019 for 1998. Table 4 gives the numbers that correspond to this plot. Table 4 has the same format as Table 2. All the mean directional differences are fairly small. The lowest correlation is in August. Only February shows a concentration below 5. Figures 7 and 8 show the mean wave directions for July and August, 1998. Both these months have concentrations below 5.0 but the correlation for August is lower. Note that the August comparisons show the low correlation as predicted by the statistics.

Table 4. Buoy 42019

Year	Month	$\overline{X}$ (deg)	$\overline{X}$ -95% confidence (deg)	$\overline{X}$ +95% confidence (deg)	$\hat{k}$	Number	Cor	Circor	Resultant	Variance	Standard deviation
1998	1	1.31	5.59	-2.97	5.07	152	0.824	0.73	0.895	0.105	0.458
1998	2	-0.11	2.85	-3.08	3.49	488	0.855	0.716	0.842	0.158	0.562
1998	3	-3.1	-1.27	-4.92	6.63	622	0.802	0.758	0.921	0.079	0.397
1998	4	1.3	3.33	-0.73	6.94	477	0.895	0.86	0.925	0.075	0.388
1998	5	2.66	4.13	1.2	17.77	340	0.763	0.761	0.971	0.029	0.239
1998	6	-0.04	1.01	-1.08	19.43	609	0.815	0.812	0.974	0.026	0.228
1998	7	3.84	4.91	2.77	47.46	235	0.907	0.907	0.989	0.011	0.146
1998	8	10.93	15.91	5.95	5.53	102	0.647	0.633	0.905	0.095	0.437
1998	9	9.65	11.8	7.5	6.25	476	0.817	0.781	0.916	0.084	0.409
1998	10	11.46	13.28	9.65	7.23	569	0.845	0.83	0.928	0.072	0.379
1998	11	5.95	8.22	3.68	5.61	481	0.896	0.835	0.906	0.094	0.434
1998	12	5.07	7.01	3.13	8.62	414	0.954	0.869	0.94	0.06	0.346

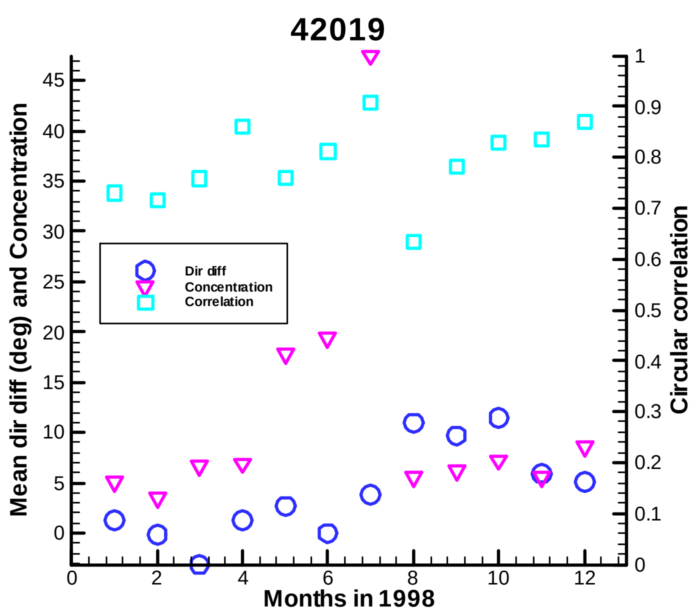

Figure 6. Statistics for 42019 and WIS for 1998. Plotting is similar to Figure 2.

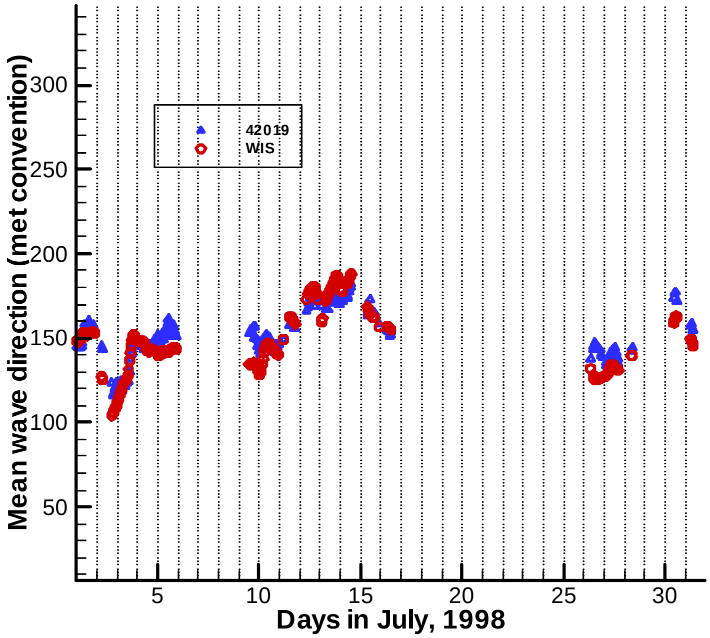

Figure 7. Measured and WIS wave directions for 42019 for July, 1998. Wave directions for waves greater than or equal to 1 m are shown.

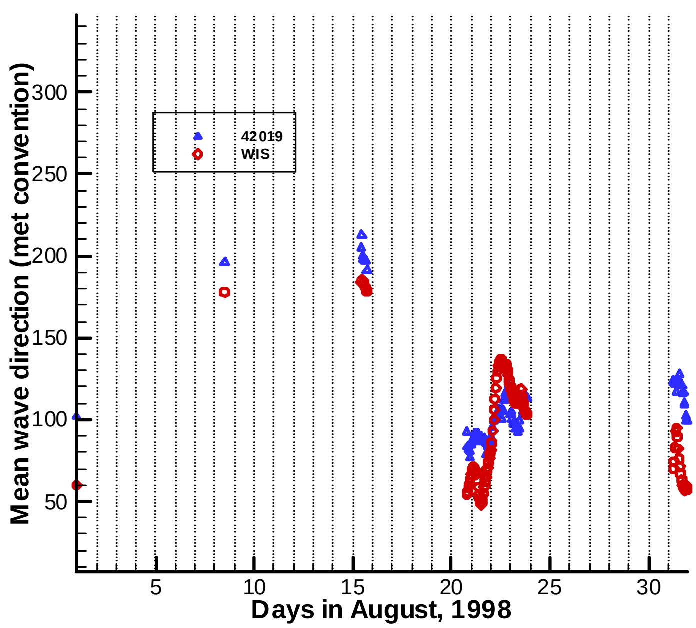

Figure 8. Measured and WIS wave directions for 42019 for August, 1998. Only waves greater than or equal to 1 m are shown.

5. SUMMARY

The statistical analysis described in this paper provides numbers to assess the quality of the WIS wave directional information in comparison to the measured information using circular statistics. Sites with large mean directional differences, low concentration statistics, and low correlations need to be checked in regard to hindcasting problems or measurement problems. The statistics provided in this paper verify that WIS mean wave directions are a good representation of the measured mean wave conditions in the Gulf of Mexico during the 1990's for the sites presented in this paper. The mean wave direction is only one parameter that describes the whole energy spectrum at each hour and is a stable quantity to use in comparisons. The analysis described in this paper could be applied to other directional parameters such as peak wave direction, etc. The mean wave direction and other directional parameters are useful in many applications, but full spectra give a better description of energy-direction characteristics. Full spectra are available for more complex applications. The statistical procedures described in this paper could be applied to calculate statistics for only high wave heights by using a 2-m or higher wave height threshold. It would also be interesting to calculate statistics for waves with directions coming from the northeastern quadrant and compare these statistics with one of the other directional quadrants.

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